3rd Annual Lexington Mathematical Tournament
Theme Round

May 5, 2012

1 Dragons

In honor of the Year of the Dragon, we present to you dragons: good and bad dragons alike.

1. It is said that Dragonite can circle the globe in just sixteen hours. If this is true, and if the Earth is assumed to be a sphere with radius 4000 miles, then what is Dragonite's average speed, in miles per hour? Use 3.14 for π and assume that the altitude above the Earth's surface at which Dragonite flies is negligible.

2. Toothless the fish-loving dragon is at (0,0) and wants to get back to his home, located at (6,6). There is a pond full of fish at (2,5). If Toothless decides to only fly east one unit or north one unit at a time, then how many paths can he take that will allow him to catch some fish before returning home?

3. Smaug the Magnificent likes to steal treasure for his ever-increasing mountain of riches. Fortunately for him, a village near his home has 2012 pieces of treasure. On the first day, he steals one piece of treasure from the village. After that, on the kth day, where $k \geq 2$, he plunders k more pieces than the amount he stole the day before. At this rate, Smaug will have stolen all of the treasure on the nth day. Find n.

4. Temeraire the studious dragon has three distinct math books, two distinct science books, and one history book that he wants to read in the next few months. If he decides not to read two books of the same subject consecutively, then in how many different orders can he read the six books?

5. Saphira the dragon is on forbidden ground patrolled by her enemy Thorn. Sensing that Thorn is 50 miles east of her and flying west at 50 miles per hour, Saphira flies north at a fast and constant speed. If Thorn can sense Saphira's presence when the two dragons are less than 40 miles away from each other, then what is the minimum speed at which Saphira must fly in order to evade his detection?

2 Knights and Knaves

In the game Knights and Knaves, each player is assigned the role of either a knight or a knave. A knight always tells the truth while a knave always lies.

6. Five players are to be assigned roles such that there are more knights than there are knaves. In how many ways can this be done?

7. Ali, Bob, Cam, Dan, and Eve play Knights and Knaves and have the following conversation. Who is (are) the knave(s)? (Note: Either includes both. For example, if both Bob and Dan are knaves, then Ali is still correct.)
   Ali: Either Bob or Dan is a knave.
   Bob: Either Ali or Eve is a knight.
   Cam: Either Bob or Eve is a knave.
   Dan: Either Ali or Cam is a knave.
   Eve: Exactly two of us are knaves.

8. Eight players sit evenly around a circular table. In how many ways can the players be assigned roles such that each player can say, "The player sitting directly across from me is a knight"?

9. Sixteen players stand in a four-by-four grid and each players says, "Every player adjacent to me (but not necessarily those diagonally across from me) is a knave." What is the minimum possible number of knights in the group?

10. Eighteen players stand in two rows of nine, one row strictly behind the other. In how many ways can the players be assigned roles such that each player can say, "At least two of the players adjacent to me (i.e. beside me in the same row or directly across from me in the other row) are knaves"?

3 Evaluate-athon

Athletes have the marathon and the triathlon. Mathletes, on the other hand, have the integrate-athon and the evaluate-athon. Since most middle-schoolers cannot integrate because they do not know calculus, we present to you the evaluate-athon. Good luck!

11. Evaluate $3 + 7 + 11 + 15 + \cdots + 51$.

12. Evaluate $123 \cdot 357 + 123 \cdot 644 + 432 \cdot 357 + 432 \cdot 644$.

13. Evaluate $\lfloor \frac{2012^4}{2011^2} \rfloor$ (where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x).

14. Evaluate $\binom{10}{10}(\frac{1}{2})^0 + \binom{10}{8}(\frac{1}{2})^2 + \binom{10}{6}(\frac{1}{2})^4 + \cdots + \binom{10}{0}(\frac{1}{2})^{10}$.

15. Evaluate $\frac{1^{-3}+3^{-3}+5^{-3}+7^{-3}+\cdots}{1^{-3}+2^{-3}+3^{-3}+4^{-3}+\cdots}$.